Human capital and the macroeconomy in an ageing society

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Introduction (1)

- Research question: How does the demographic structure of the population affect the long-run macroeconomic performance of an economy?

	1950	1960	1970	1980	1990	2000	2005
United States	2.4	2.2	1.6	1.5	1.5	1.4	1.1
	68.6	70.2	71.5	74.5	75.6	77.2	78.0
	14.3	17.5	18.7	19.8	21.3	21.0	20.5
Japan	2.4	1.7	1.9	1.3	1.0	0.9	0.9
	62.2	69.0	73.1	77.0	79.5	81.8	82.7
	10.0	10.6	11.7	15.0	19.4	27.6	32.5
United Kingdom	1.5	1.8	1.4	1.3	1.3	1.1	1.2
	69.3	71.0	72.2	74.1	76.2	78.4	79.6
	17.9	20.2	23.3	26.8	26.9	26.8	26.9
Netherlands	2.2	2.1	1.5	1.2	1.3	1.2	1.1
	71.9	73.5	74.1	76.1	77.3	78.7	80.2
	14.0	16.8	18.8	20.0	20.8	21.9	22.8

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Introduction (2)

- Objective of this paper: Study the macroeconomic general equilibrium effects of demographic shocks.
  - Increase in longevity.
  - Decrease in the birth rate.
- Model features:
  - Closed economy (ageing is a global phenomenon).
  - Overlapping generations of finitely-lived agents.
  - Accumulation of both physical and human capital.
  - Distinction between economic ageing and biological ageing.
  - Simple exogenous growth mechanism.
- Main results:
  - Agents respond to a biological longevity boost by schooling a little more and working a bit longer. Most of the increase in the length of life is consumed in the form of leisure.
  - A comprehensive longevity boost increases the length of the work career dramatically.
  - The quantitative effects of a baby bust are rather small.
- The main results are robust to imperfections in the capital market (borrowing constraints) and the labour market (indivisible labour).

Outline

Model: Producers

- Representative firm that produces an aggregate output $Y(t)$.
- Linear homogeneous production technology:

$$Y(t) = F(K(t), Z(t)HC(t))$$

  where:
  - $K(t)$ is the stock of physical capital.
  - $HC(t)$ is the amount of human capital employed.
  - $Z(t) = Ze^{gt}$ is an index of Harrod-neutral technological progress with $g \geq 0$ the exogenous growth rate.
- Profit maximization gives the usual marginal productivity conditions:

$$r(t) + \delta_k = \frac{\partial F(K(t), Z(t)HC)}{\partial K(t)}$$
$$\frac{w(t)}{Z(t)} = \frac{\partial F(K(t), Z(t)HC)}{\partial [Z(t)HC]}$$

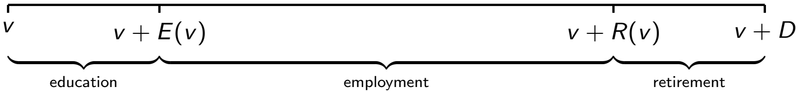
  where:
  - $r(t)$ is the real interest rate.
  - δ_k is the depreciation rate of physical capital.
  - $w(t)$ is the rental rate of human capital.

Model: Consumers (1)

- Overlapping generations of finitely-lived consumers.
- Life-time utility of an agent of vintage v:

$$\Lambda(v) = \int_{v}^{v+D} \Phi(c(v,t), z(v,t)) e^{-\rho(t-v)} dt$$

  where:
  - $\Phi(\cdot)$ is the instantaneous utility function.
  - $c(v,t)$ is consumption at time t.
  - $z(v,t)$ is leisure at time t.
  - ρ is the pure rate of time preference.
  - D is the age at certain death ('rectangular mortality', no longevity risk).
- Schematic overview of the life cycle:



Model: Consumers (2)

- The accumulation of financial assets $a(v,t)$ proceeds according to:

$$\dot{a}(v,t) = r(t)a(v,t) + w(t)h(v,t)l(v,t) - c(v,t)$$

where:

  - $\dot{a}(v,t) \equiv \partial a(v,t)/\partial t$ is net asset accumulation.
  - $l(v,t)$ is labour supply.
  - $h(v,t)$ is the stock of human capital.

- Under the assumption of perfect capital markets, the life-time budget constraint can be written as:

$$\int_{v+E(v)}^{v+R(v)} w(t)h(v,t)l(v,t)e^{-\int_v^t r(\tau)d\tau}dt = \int_v^{v+D} c(v,t)e^{-\int_v^t r(\tau)d\tau}dt$$

Model: Consumers (3)

- The accumulation of human capital $h(v,t)$ proceeds according to:

$$\frac{\dot{h}(v,t)}{h(v,t)} = \underbrace{\gamma l(v,t)}_{\text{experience}} - \underbrace{\delta_h(t-v)}_{\text{depreciation}} \quad \text{for } E(v) \le t-v \le D, \quad \underbrace{h(v, v+E(v)) = e^{G(E(v))}}_{\text{education}}$$

  where:
  - $\gamma > 0$ is a measure of returns to experience.
  - $\delta_h(t-v)$ is age-dependent rate of human capital depreciation.

- Marginal effects:

$$\frac{\partial h(v,t)}{\partial E(v)} = \Big[G'(E(v)) - [\gamma l(v, v+E(v)) - \delta_h(E(v))]\Big] h(v,t)$$

$$\frac{\partial h(v,t)}{\partial l(v,\tau)} = \begin{cases} \gamma h(v,t) & \text{for } \tau < t \\ 0 & \text{for } \tau \ge t \end{cases}$$

- Formal education and working are mutually exclusive activities. The cost of education is e_0 units of time during the schooling period.

Model: Consumers (4)

- The problem of the consumer is to choose $E(v)$, $R(v)$, $\{c(v,t)\}_{t=v}^{v+D}$ and $\{l(v,t)\}_{t=v+E(v)}^{v+R(v)}$ to maximize life-time utility subject to the budget constraint and the process of human capital accumulation.
- First-order condition with respect to consumption:

$$\Phi_c(c(v,t),z(v,t))e^{-\rho(t-v)} = \lambda(v)e^{-\int_v^t r(\tau)d\tau}$$

- First-order condition with respect to labour supply:

$$\Phi_z(c(v,t),z(v,t))e^{-\rho(t-v)} = \lambda(v)\left[w(t)h(v,t) + \gamma V_H(v,t)\right]e^{-\int_v^t r(\tau)d\tau}$$

where $V_H(v,t)$ is the discounted value of future wage income:

$$V_H(v,t) = \int_t^{v+R(v)} w(\tau)h(v,\tau)l(v,\tau)e^{-\int_t^\tau r(s)ds}d\tau$$

The opportunity cost of time consists of both the current wage rate $w(t)h(v,t)$ (backward-looking) and the imputed value of foregone experience $\gamma V_H(v,t)$ (forward-looking).

Model: Consumers (5)

- First-order condition with respect to education.

$$
\begin{aligned}
0 = {} & \underbrace{\left[\Phi(c(v, v+E(v)^{-}), 1-e_0) - \Phi(c(v, v+E(v)), 1-l(v, v+E(v)))\right] e^{-\rho E(v)}}_{\text{utility effect}} \\
& + \underbrace{\lambda(v)\left[c(v, v+E(v)) - c(v, v+E(v)^{-})\right] e^{-\int_v^{v+E(v)} r(\tau)d\tau}}_{\text{consumption effect}} \\
& - \underbrace{\lambda(v) w(v+E(v)) h(v, v+E(v)) l(v, v+E(v)) e^{-\int_v^{v+E(v)} r(\tau)d\tau}}_{\text{earnings effect}} \\
& + \underbrace{\lambda(v)\left\{G'(E(v)) - [\gamma l(v, v+E(v)) - \delta_h(E(v))]\right\} V_H(v, v+E(v)) e^{-\int_v^{v+E(v)} r(\tau)d\tau}}_{\text{human capital effect}}
\end{aligned}
$$

- First-order condition with respect to retirement age:

$$
l(v, v+R(v)^{-}) = 0
$$

Model: Demography

- The size of the population at time t is $P(t)$.
- Agents are fertile for ages $0 \leq s \leq B < D$ and the net birth rate b is age-independent.
- The size of the cohort born at time v is $P(v, v) = b \int_{v-B}^{v} P(s, v) dv$.
- Demographic steady state: the population grows at a constant rate n.
- Demographic equilibrium condition:

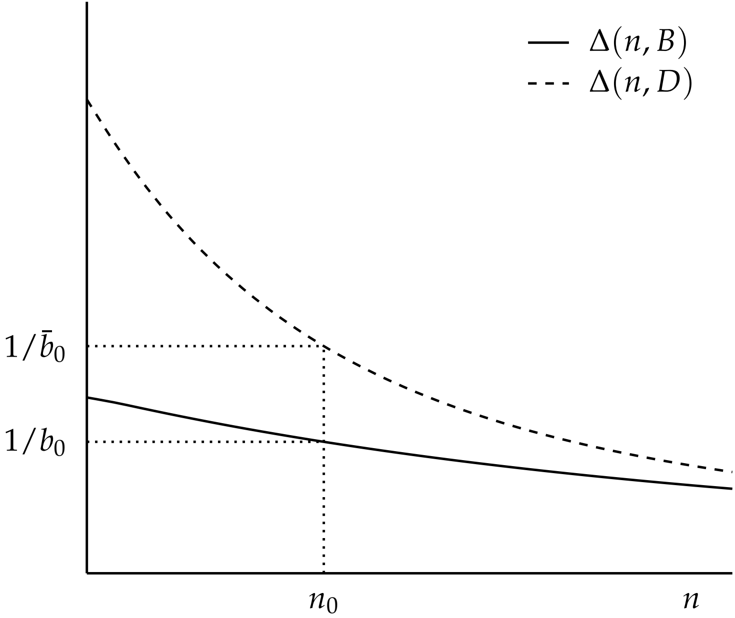
$$1 = b\Delta(n, B), \qquad \text{with} \quad \Delta(\theta, T) = \begin{cases} \dfrac{1 - e^{-\theta T}}{\theta} & \text{if } \theta \neq 0 \\ T & \text{if } \theta = 0 \end{cases}$$

- Relative cohort sizes:

$$p\left(v, v+u\right) \equiv \begin{cases} \bar{b}e^{-nu} & \text{for } 0 \leq u \leq D \\ 0 & \text{for } u > D \end{cases}, \qquad \bar{b} \equiv \frac{P(t,t)}{P(t)} = \frac{1}{\Delta\left(n, D\right)}$$

- Given the demographic structure of the population we can calculate aggregate human capital $HC(t)$, consumption $C(t)$, and financial assets $A(t)$.

Demographic equilibrium



Model: Macroeconomic equilibrium

- Equilibrium in the financial market implies that $A(t) = K(t)$.
- Without proving uniqueness of the equilibrium we can show that there exists a balanced growth path, given some restrictions on the functional form of the instantaneous utility function (King-Plosser-Rebelo condition).
- By scaling individual and aggregate variables appropriately we can obtain stationary values (in case of macro variables) or age-profiles (in case of micro choices).
  - Micro:

$$\tilde{c}(u) \equiv \frac{c(v, v+u)}{Z(v)}, \quad \tilde{a}(u) \equiv \frac{a(v, v+u)}{Z(v)}, \quad \tilde{V}_H(u) \equiv \frac{V_H(v, v+u)}{Z(v)}$$

$$l(u) \equiv l(v, v+u), \quad h(u) \equiv h(v, v+u), \quad E, \quad R$$

  - Macro:

$$hc \equiv \frac{HC(t)}{P(t)}, \quad k \equiv \frac{K(t)}{Z(t)P(t)}, \quad c \equiv \frac{C(t)}{Z(t)P(t)}, \quad y \equiv \frac{Y(t)}{Z(t)P(t)}$$

$$\tilde{w} \equiv \frac{w(t)}{Z(t)}, \quad r$$

Calibration: Functional forms (1)

- Instantaneous utility function:

$$\Phi(c,z) = \begin{cases} \dfrac{\left[c^{1-\varepsilon_z} z^{\varepsilon_z}\right]^{1-1/\sigma} - 1}{1-1/\sigma} & \text{if } \sigma > 0, \sigma \neq 1 \\ (1-\varepsilon_z)\ln c + \varepsilon_z \ln z & \text{if } \sigma = 1 \end{cases}$$

- Return to education:

$$G(E) = \beta_0 + \beta_1 E - \beta_2 E^2, \quad \beta_1 > 0, \quad \beta_2 > 0$$

- Human capital depreciation:

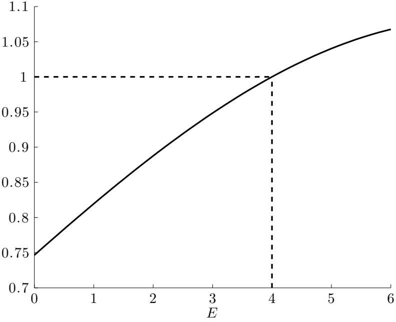
$$\delta_h(u) = \delta_0 + \frac{1}{\delta_1(\bar{R}-u)} \quad \text{for } 0 \leq u < \bar{R}, \quad \delta_0 > 0, \quad \delta_1 > 0$$

- Production function:

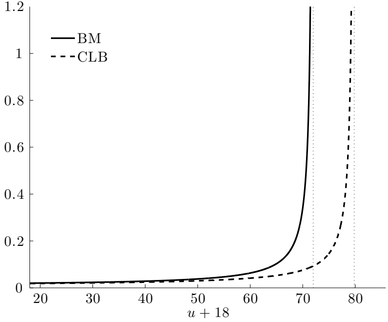
$$F(K(t), Z(t)H(t)) = \Omega_0 K(t)^{\varepsilon_k} [Z(t)H(t)]^{1-\varepsilon_k}, \quad \Omega_0 > 0, \quad 0 < \varepsilon_k < 1$$

Calibration: Functional forms (2)

(a) Initial stock $e^{G(E)}$



(b) Depreciation rate $\delta_h(u)$



Parameter values

- We first choose calibration targets at the micro and macro level:

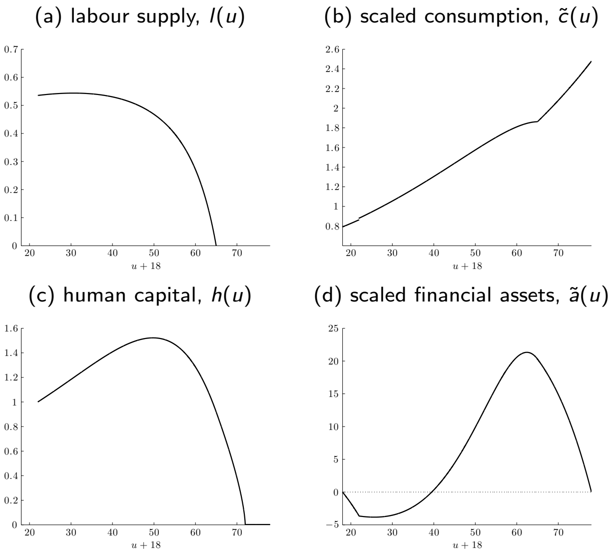
$$E = 4\ (22), \quad R = 47\ (65), \quad y = 1, \quad r = 0.05$$

- Then we set the value of some parameters for which we have a good intuition and/or microeconometric evidence:

$$\bar{R} = 54\ (72), \quad D = 60\ (78), \quad \gamma = 0.08, \quad n = 0.01, \quad g = 0.02, \quad \ldots$$

- The values of the remaining parameters are implied.

Benchmark model: Age profiles (1)



Benchmark model: Age profiles (2)

Demographic shocks (1)

- Benchmark model (BM)

$$D = 60, \quad \bar{R} = 54, \quad \bar{b}\ (\times 100\%) = 2.22, \quad n\ (\times 100\%) = 1.00$$

- Biological longevity boost (BLB)

$$D = 67.8, \quad \bar{R} = 54, \quad \bar{b}\ (\times 100\%) = 2.03, \quad n\ (\times 100\%) = 1.00$$

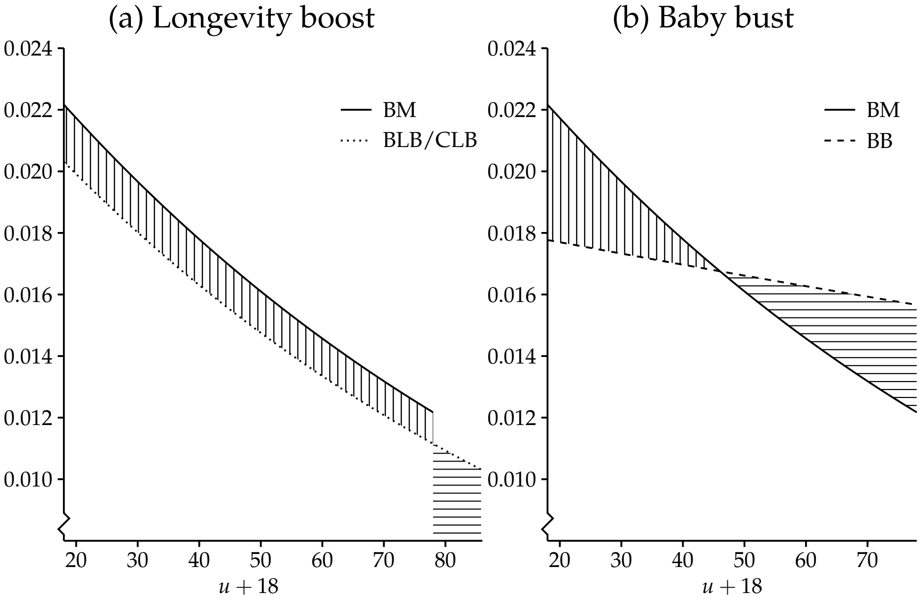
- Comprehensive longevity boost (CLB)

$$D = 67.8, \quad \bar{R} = 61.8, \quad \bar{b}\ (\times 100\%) = 2.03, \quad n\ (\times 100\%) = 1.00$$

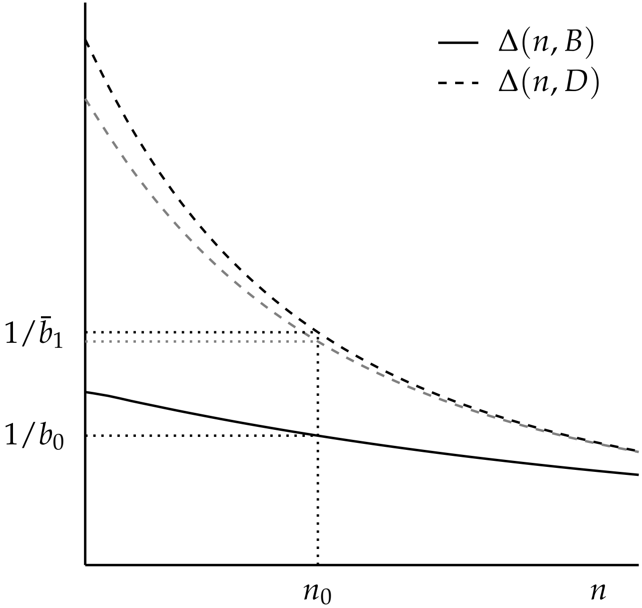
- Baby bust (BB)

$$D = 60, \quad \bar{R} = 54, \quad \bar{b}\ (\times 100\%) = 1.77, \quad n\ (\times 100\%) = 0.21$$

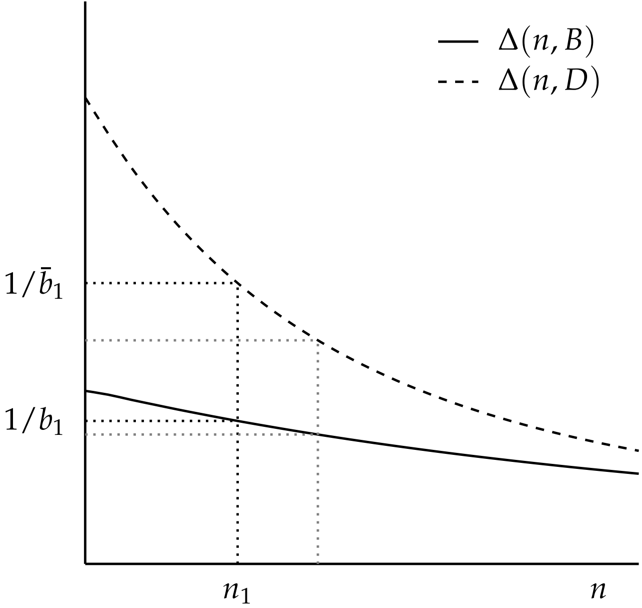
Demographic shocks (2a)



Demographic shocks (2b): Longevity boost



Demographic shocks (2c): Baby bust



Demographic shocks (3)

	BM	*BLB*		*CLB*		*BB*
Micro choices						
E	4.000	4.121	4.395	4.426	4.146	4.114
R	47.000	47.602	48.878	55.409	53.550	47.575
$\tilde{c}(0)$	0.791	0.797	0.897	0.945	0.832	0.829
$l(E)$	0.535	0.535	0.536	0.535	0.534	0.532
$h(E)$	1.000	1.006	1.020	1.021	1.008	1.006
$h(R)$	0.919	0.926	0.877	1.115	1.165	0.901
$e^{-rE}\tilde{V}_H(E)$	22.572	23.737	27.148	28.512	24.733	23.773
$\Lambda(v_0)$	-11.591	-11.562	-11.514	-10.683	-10.807	-11.559
Macro outcomes						
y	1.000		1.044		1.159	0.990
k	1.960		2.113		2.207	1.965
hc	0.454		0.470		0.531	0.448
c	0.804		0.833		0.938	0.809
$\tilde{w}$	1.684		1.701		1.669	1.690
r ($\times 100\%$)	5.000		4.615		5.350	4.850

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$e^{-rE}\tilde{V}_H(E)$	**22.572**	23.737	27.148	**28.512**	24.733	23.773
$\Lambda(v_0)$	**−11.591**	−11.562	−11.514	**−10.683**	−10.807	−11.559
Macro outcomes						
y	**1.000**		1.044		1.159	0.990
k	**1.960**		2.113		2.207	1.965
hc	**0.454**		0.470		0.531	0.448
c	**0.804**		0.833		0.938	0.809
$\tilde{w}$	**1.684**		1.701		1.669	1.690
r ($\times 100\%$)	**5.000**		4.615		5.350	4.850

Demographic shocks (3)

	BM	*BLB*		*CLB*		*BB*
Micro choices						
E	**4.000**	4.121	4.395	4.426	**4.146**	4.114
R	**47.000**	47.602	48.878	55.409	**53.550**	47.575
$\tilde{c}(0)$	**0.791**	0.797	0.897	0.945	**0.832**	0.829
$l\,(E)$	**0.535**	0.535	0.536	0.535	**0.534**	0.532
$h\,(E)$	**1.000**	1.006	1.020	1.021	**1.008**	1.006
$h\,(R)$	**0.919**	0.926	0.877	1.115	**1.165**	0.901
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Demographic shocks (4)

- Biological longevity boost (BLB).
  - Small increase in E, R and $R - E$, large increase in $D - R$.
  - Rise in human capital is dominated by increase in physical capital: r decreases and $\tilde{w}$ increases.
  - Output rises.
- Comprehensive longevity boost (CLB).
  - Small increase in E, large increase in R and $R - E$, decrease in $D - R$.
  - Rise in human capital dominates increase in physical capital: r increases and $\tilde{w}$ decreases.
  - Output rises.
- Baby bust (BB).
  - Small increase in E, R and $R - E$.
  - Fall in human capital, rise in physical capital: r decreases and $\tilde{w}$ increases.
  - Output falls.

Robustness checks

- The main results still hold if we change some of the assumptions underlying the benchmark model.
  - Borrowing constraints and perfect study loan system.

$$a(v, t) \geq 0$$

  - Indivisible labour.

$$l(v, t) \in \{0, l_F\}$$

Robustness checks: Borrowing constraints (1)

- In reality people might not be able to borrow: $a(v,t) \geq 0$.
- If there are no study loans then the (trivial) solution is $E(v) = 0$. Output falls substantially.
- Suppose there are perfect study loans.
  - A person can borrow during the schooling period but has to start repaying immediately afterwards:

$$\dot{s}(v,t) = \begin{cases} r(t)s(v,t) + c(v,t) & \text{for } 0 \leq t - v < E(v) \\ r(t)s(v,t) - i(v) & \text{for } E(v) \leq t - v \leq D \end{cases}$$

$$s(v,v) = s(v, v+D) = 0$$

    where $i(v)$ is the redemption payment.
  - Regular assets satisfy:

$$\dot{a}(v,t) = r(t)a(v,t) + w(t)h(v,t)l(v,t) - c(v,t) - i(v) \quad \text{for } E(v) \leq t - v \leq D$$

$$a(v, v+E(v)) = a(v, v+D) = 0, \quad a(v,t) \geq 0$$

Robustness checks: Borrowing constraints (2)

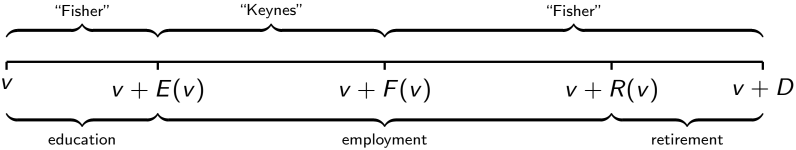
- Two regimes can be distinguished.
  - "Fisher" regime: Borrowing constraint is not binding, consumption and leisure are limited by a present-value constraint, e.g. for $t \geq v + F(v)$:

$$\int_t^D \left[c(v,\tau)+i(v)\right] e^{-\int_t^\tau r(s)ds} d\tau = a(v,t)+\int_t^D w(\tau)h(v,\tau)l(v,\tau)e^{-\int_t^\tau r(s)ds} d\tau$$

  - "Keynes" regime: Borrowing constraint is binding, consumption and leisure are limited by a flow constraint.

$$c(v,t) = w(t)h(v,t)l(v,t) - i(v)$$

- Let $F(v)$ denote the age at which the borrowing constraint ceases to bind.
- Schematic overview of the life cycle:



Robustness checks: Borrowing constraints (3)

	BM	*BC*		*BLB*	*CLB*	*BB*
Micro choices						
E	**4.000**	3.988	4.032	4.496	4.245	4.173
F		7.815	8.396	11.979	13.490	10.211
R	**47.000**	46.873	47.083	49.061	53.926	47.712
$\tilde{c}(0)$	**0.791**	0.785	0.798	0.923	0.856	0.842
$\tilde{i}$		0.192	0.196	0.225	0.227	0.206
$l(E)$	**0.535**	0.535	0.549	0.561	0.559	0.551
$h(E)$	**1.000**	0.999	1.002	1.025	1.013	1.009
$h(R)$	**0.919**	0.920	0.913	0.859	1.134	0.890
$e^{-rE}\tilde{V}_H(E)$	**22.572**	22.433	22.874	28.358	26.004	24.381
$\Lambda(v_0)$	**−11.591**	−11.593	−11.583	−11.497	−10.776	−11.541
Macro outcomes						
y	**1.000**		1.001	1.047	1.163	0.993
k	**1.960**		1.972	2.154	2.253	1.989
hc	**0.454**		0.454	0.469	0.530	0.448
c	**0.804**		0.804	0.831	0.937	0.809
$\tilde{w}$	**1.684**		1.686	1.709	1.678	1.695
r ($\times 100\%$)	**5.000**		4.938	4.427	5.133	4.734

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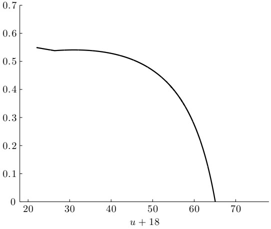
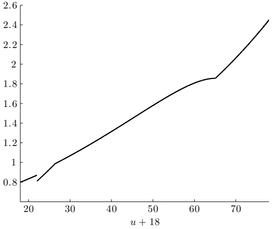
Robustness checks: Borrowing constraints (4)

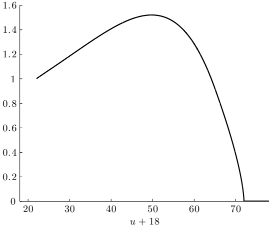
(a) labour supply, $l(u)$



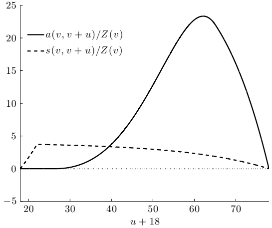
(b) scaled consumption, $\tilde{c}(u)$



(c) human capital, $h(u)$



(d) scaled assets and study debt, $\tilde{a}(u)$ and $\tilde{s}(u)$



Demographic shocks: Comparing BM and BC

	BLB			CLB			BB		
	BM	**BC**	IL	BM	**BC**	IL	BM	**BC**	IL
$E^{\ddagger}$	0.40	**0.46**	0.34	0.15	**0.21**	0.12	0.11	**0.14**	0.20
$F^{\ddagger}$		**3.58**			**5.09**			**1.82**	
$R^{\ddagger}$	1.88	**1.98**	3.23	6.55	**6.84**	6.22	0.58	**0.63**	0.88
$(R-E)^{\ddagger}$	1.48	**1.51**	2.89	6.40	**6.63**	6.10	0.46	**0.49**	0.68
$(D-R)^{\ddagger}$	5.92	**5.82**	4.58	1.25	**0.96**	1.58	−0.58	**−0.63**	−0.88
$y^{\sharp}$	4.39	**4.69**	−0.54	15.92	**16.25**	13.17	−0.97	**−0.75**	−1.21
$k^{\sharp}$	7.85	**9.23**	3.48	12.63	**14.24**	10.38	0.29	**0.02**	0.15
$hc^{\sharp}$	3.35	**3.16**	−1.74	16.95	**16.68**	14.04	−1.35	**−0.01**	−1.58
$r^{\S}$	−38.85	**−51.16**	−46.89	34.99	**19.47**	30.57	−14.99	**−20.41**	−16.00
$\tilde{w}^{\sharp}$	1.01	**1.36**	1.22	−0.88	**−0.50**	−0.77	0.39	**0.01**	0.41

Notes: ‡change in years; $^{\sharp}$percentage change; §change in base points.

Robustness checks: Indivisible labour (1)

- In reality people might not be able to choose hours in the labour market (intensive margin) but only participation (extensive margin).
- Agents start working full-time at age $E(v)$ until they leave the labour market at age $R(v)$.

$$l(v,t) = l_F \quad \text{for } E(v) \leq t - v < R(v)$$

- The retirement age has to satisfy:

$$\begin{aligned} 0 = & \left[\Phi(c(v, v + R(v)^{-}), 1 - l_F) - \Phi(c(v, v + R(v)), 1)\right] e^{-\rho R(v)} \\ & - \lambda e^{-\int_v^{v+R(v)} r(\tau) d\tau} \left[c(v, v + R(v)^{-}) - c(v, v + R(v))\right] \\ & + \lambda e^{-\int_v^{v+R(v)} r(\tau) d\tau} w(v + R(v)) h(v, v + R(v)) l_F \end{aligned}$$

Robustness checks: Indivisible labour (2)

	BM	*IL*		*BLB*	*CLB*	*BB*
Micro choices						
E	**4.000**	3.848	3.787	4.127	3.903	3.898
R	**47.000**	42.265	41.776	45.001	47.992	42.658
$\tilde{c}(0)$	**0.791**	0.748	0.730	0.807	0.757	0.763
$l(E)$	**0.535**	0.500	0.500	0.500	0.500	0.500
$h(E)$	**1.000**	0.992	0.989	1.007	0.995	0.995
$h(R)$	**0.919**	1.305	1.317	1.206	1.593	1.294
$e^{-rE}\tilde{V}_H(E)$	**22.572**	21.208	20.638	24.132	22.246	21.692
$\Lambda(v_0)$	**−11.591**	−11.749	−11.767	−11.707	−10.991	−11.733
Macro outcomes						
y	**1.000**		0.936	0.931	1.060	0.925
k	**1.960**		1.821	1.885	2.011	1.824
hc	**0.454**		0.426	0.419	0.486	0.420
c	**0.804**		0.754	0.743	0.859	0.757
$\tilde{w}$	**1.684**		1.680	1.700	1.667	1.687
r ($\times$100%)	**5.000**		5.087	4.618	5.393	4.927

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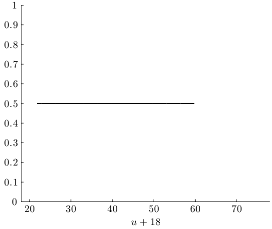
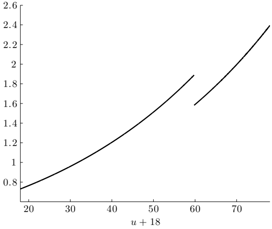
Robustness checks: Indivisible labour (3)

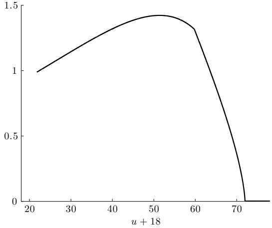
(a) labour supply, $l(v)$



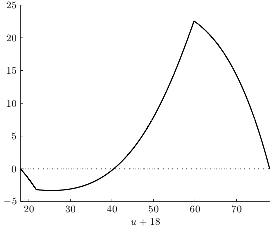
(b) scaled consumption, $\tilde{c}(u)$



(c) human capital, $h(u)$



(d) scaled financial assets, $\tilde{a}(u)$



Demographic shocks: Comparing BM and IL

	BLB			CLB			BB		
	BM	BC	**IL**	BM	BC	**IL**	BM	BC	**IL**
$E^{\ddagger}$	0.40	0.46	**0.34**	0.15	0.21	**0.12**	0.11	0.14	**0.20**
$F^{\ddagger}$		3.58			5.09			1.82	
$R^{\ddagger}$	1.88	1.98	**3.23**	6.55	6.84	**6.22**	0.58	0.63	**0.88**
$(R-E)^{\ddagger}$	1.48	1.51	**2.89**	6.40	6.63	**6.10**	0.46	0.49	**0.68**
$(D-R)^{\ddagger}$	5.92	5.82	**4.58**	1.25	0.96	**1.58**	−0.58	−0.63	**−0.88**
$y^{\sharp}$	4.39	4.69	**−0.54**	15.92	16.25	**13.17**	−0.97	−0.75	**−1.21**
$k^{\sharp}$	7.85	9.23	**3.48**	12.63	14.24	**10.38**	0.29	0.02	**0.15**
$hc^{\sharp}$	3.35	3.16	**−1.74**	16.95	16.68	**14.04**	−1.35	−0.01	**−1.58**
$r^{\S}$	−38.85	−51.16	**−46.89**	34.99	19.47	**30.57**	−14.99	−20.41	**−16.00**
$\tilde{w}^{\sharp}$	1.01	1.36	**1.22**	−0.88	−0.50	**−0.77**	0.39	0.01	**0.41**

Notes: ‡change in years; $^{\sharp}$percentage change; §change in base points.

Conclusion

- We study the steady-state general equilibrium effects of a number of stylized ageing shocks that have hit most western countries over the last half century.
- Our results are based on a plausibly calibrated version of our micro-founded theoretical model. They are robust to alternative modeling assumptions.
- We find that it is crucial to distinguish between economic and biological longevity.
- Directions for future research:
  - Empirical embedding of the human capital accumulation process.
  - Introduction of stochastic shocks in the labour market.
  - Introduction of a pension system.